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Direct Torque Control of Three Phase Induction Motors: Concept & Principles

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Abstract— This paper thoroughly illustrates the Direct Torque Control (DTC) concept, principles and theory. It describes the dynamic behavior of a direct torque controlled three-phase induction motor. The complete analysis begins with a suitable mathematical model of the motor and the inverter circuit. It represents the DTC main parts and describes its operation. This paper also illustrates how the DTC can become a base for speed control schemes and the ability to switch between torque & speed control.

Keywords: *Direct Torque Control, DTC, Flux Vector Control, 3-phase Induction Motor, Motor Control, Variable Speed Drive*

1. INTRODUCTION

Direct Torque Control (DTC) is one of the latest developments in ac motor control. It provides high torque dynamic response. DTC almost re-establishes dc drive advantages through direct torque and flux control implementation, which electrical engineers and researchers were looking for. Since its introduction in 1985, the DTC principle has been widely applied to fast dynamic induction motor drives. Despite DTC's simplicity, it can produce very fast flux and torque control. And if the flux and torque are accurately estimated, DTC is almost unaffected by motor parameters and perturbations. However, notable flux, torque and current pulsations occur during steady-state motor operation [1].

Regarding induction motor control, the intention is directed to control its output quantities, namely torque and speed. Induction motor speed control is more famous than torque control. However, industrial applications need torque control as well as speed control in some cases. Also, torque control can be used as a base to speed control.

2. THREE-PHASE INDUCTION MOTORS MATHEMATICAL MODEL

The space vector concept, also called space phasor, has been used in the ac motor drives analysis since it is more suitable for investigating the dynamic behaviour of the motor. The basic idea of this concept is to transform the instantaneous three-phase machine variables such as voltages, fluxes and currents to space vectors onto a complex plane located in the motor cross-section. In this plane, the space phasor rotates with an angular speed equal to the angular frequency of the three-phase supply with respect to the fixed (stationary) reference frame [2, 1].

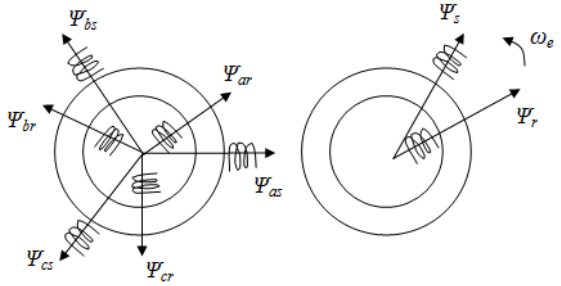

Fig. 1. Space phasor of induction motor rotating fields

Fig. 1 illustrates how the flux linkage space vector, which rotates in the machine space along the air gap periphery, represents the three-phase time-varying fluxes.

2.1 Dynamic Model in Space Vector Form

For a three-phase induction motor, the space vector Y of the stator current, Flux or voltage is defined from its phase quantities by:

$$\mathbf{Y} = (2/3)\ [\ Y_a(t) + a\ .\ Y_b(t) + a^2\ .\ Y_c(t)\ ] \quad (1)$$

where $a = exp(j2\pi/3)$, note that space vectors are denoted as boldface letters.
The above transform is reversible, and each phase quantity can be calculated from the space vector by:

$$Y_a = Re\ (\mathbf{Y}),\ Y_b = Re\ (a^2.\mathbf{Y}),\ Y_c = Re\ (a.\mathbf{Y}). \quad (2)$$

where Re(**Y**) and Im(**Y**) are the real and imaginary values of a space vector **Y**.
With space vector notation, we can deduce the dynamic model and equivalent circuit of the induction motor referred to the stationary reference frame (fixed to stator) as follow [3]:
Voltage equations on the stator and rotor circuits are:

$$\boldsymbol{V_s} = R_s\ \boldsymbol{I_s} + D\ \boldsymbol{\Psi_s} \quad (3)$$
$$\boldsymbol{V_r'} = R_r'\ \boldsymbol{I_r'} + D\ \boldsymbol{\Psi_r'} = 0 \quad (4)$$

where *"D"* is the derivative operator w.r.t. time (d/dt) and $\boldsymbol{V}$, $\boldsymbol{I}$ and $\boldsymbol{\Psi}$ are motor voltage, current and flux linkage, respectively and subscripts *"s, r"* donate stator and rotor quantities. Primed quantities are stator and rotor variables referred to as their reference frames. Usually, actual rotor variables ($\boldsymbol{V_r'}$, $\boldsymbol{I_r'}$, $\boldsymbol{\Psi_r'}$) of Eq.(4), which are computed in the rotor reference frame, is transformed into new variables ($\boldsymbol{V_r}$, $\boldsymbol{I_r}$, $\boldsymbol{\Psi_r}$) referred to stator reference frame as in the conventional steady-state equivalent circuit derivation. Let the stator to rotor winding turns ratio be *"n"* and the angular position of the rotor be *"θ"*, and let us define:

$$\boldsymbol{I_r} = (1/n)\ exp(j\,\theta)\ \boldsymbol{I_r'}\ ,\quad \boldsymbol{\Psi_r} = n\ exp(j\theta)\ \boldsymbol{\Psi_r'} \quad (5)$$

where *"j"* is the complex operator. Also, by defining referred rotor impedances as $R_r = n^2\ R_r'$, $Ll_r = n^2\ Ll_r'$, we can rewrite Eq.(4) referred to stator reference frame as:

$$0 = R_r\ \boldsymbol{I_r} + (D - j\ \omega_o)\ \boldsymbol{\Psi_r} \quad (6)$$
$$0 = R_r\ \boldsymbol{I_r} + D\ \boldsymbol{\Psi_r} - j\ \omega_o\ \boldsymbol{\Psi_r} \quad (7)$$

where $\omega_o = D\ \theta_o$, is the speed of the motor in electrical frequency units, so the term ($\omega_o\ \boldsymbol{\Psi_r}$) is called speed voltage drop, which expresses the power conversion. Also, the flux linkages can be expressed as:

$$\boldsymbol{\Psi_s} = L_s\ \boldsymbol{I_s} + L_m\ \boldsymbol{I_r} \quad (8)$$
$$\boldsymbol{\Psi_r} = L_m\ \boldsymbol{I_s} + L_r\ \boldsymbol{I_r} \quad (9)$$

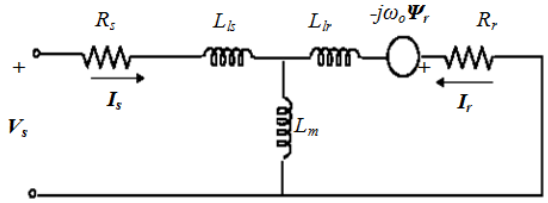

Fig. 2. Dynamic equivalent circuit referred to the stationary reference frame

The four equations Eqs.(3, 6, 8 & 9) constitute an induction motor dynamic model referred to as stationary (stator) reference frame in space phasor form. By eliminating flux linkages, model equations can be simplified as follows:

$$\boldsymbol{V_s} = (R_s + L_s\ D)\ \boldsymbol{I_s} + L_m\ D\ \boldsymbol{I_r} \quad (10)$$
$$0 = [R_r + L_r\ (D - j\ \omega_o)]\ \boldsymbol{I_r} + L_m\ (D - j\omega_o)\ \boldsymbol{I_s} \quad (11)$$

By restoring the speed voltage term in the previous equation, we obtain Eq.(12):

$$0 = (R_r + L_r D)\, \boldsymbol{I_r} + L_m D\, \boldsymbol{I_s} - j\, \omega_o\, \boldsymbol{\Psi_r} \quad (12)$$

From Eqs.(10, 12), the model of dynamic equivalent circuit referred to stationary reference frame can be drawn as in Fig. 2. With excitation frequency ω_e at steady state operation, the derivative operator D in Eqs.(10, 11) is replaced by $j\omega_e$ and after some algebraic rearrange, we will get:

$$\boldsymbol{V_s} = (R_s + j\omega_e L_s)\, \boldsymbol{I_s} + j\omega_e L_m\, \boldsymbol{I_r} \quad (13)$$
$$0 = (R_r/s + j\omega_e L_r)\, \boldsymbol{I_r} + j\omega_e L_m\, \boldsymbol{I_s} \quad (14)$$

Those entirely describe the famous and conventional steady-state equivalent circuit. The above-mentioned procedure is general and accordingly, the dynamic model may be described in any arbitrary reference frame rotating at speed ω_a. The previous analysis, the stator reference frame, is a special case of the general one with $\omega_a = 0$. In the case of the analysis referred to the rotor reference frame, we have $\omega_a = \omega_o$. In the case of the analysis with respect to the synchronously rotating reference frame, we have $\omega_a = \omega_e$ [3]. In the present case, direct torque control, the analysis with respect to the stationary reference frame is suitable and enough. Generally, the suitable choice of reference frame is significant for simplifying motor analysis and control.

2.2 d-q Equivalent Circuit

Often, induction motors analysis with a space vector model is complicated because we have to deal with complex number variables. For any space phasor or vector **Y**, two real quantities Y_q and Y_d can be defined as follows:

$$\boldsymbol{Y} = Y_q - j\, Y_d \quad (15)$$

In other words, Y_q = Re (**Y**) and Y_d = - Im (**Y**). Fig. 3 illustrates how the d-q axes are defined on a stationary reference frame at a certain angle with respect to a-b-c frame. This angle is equal to zero in our analysis (the q-axis lies on the a-axis, which is taken as a reference).

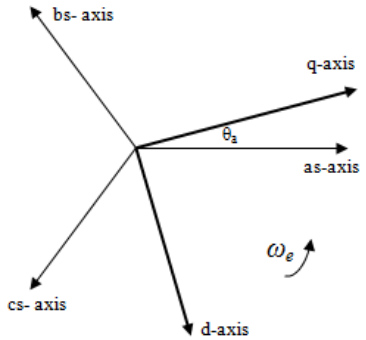

Fig. 3. d and q axes definition on an arbitrary reference frame

As mentioned before, the space vector **Y** represents voltage, current or Flux linkage. With the above definition in Eq.(15), Eqs. (10, 11) can be translated into the following four equations of real variables as follow:

$$V_{qs} = (R_s + L_s D)\, I_{qs} + L_m D\, I_{qr} \quad (16)$$
$$V_{ds} = (R_s + L_s D)\, I_{ds} + L_m D\, I_{dr} \quad (17)$$
$$0 = L_m D\, I_{qs} - \omega_o L_m I_{ds} + (R_r + L_r D)\, I_{qr} - \omega_o L_r I_{dr} \quad (18)$$
$$0 = -\omega_o L_m I_{qs} + L_m D\, I_{ds} - \omega_o L_r I_{qr} + (R_r + L_r D)\, I_{dr} \quad (19)$$

Also Eq.(12) can be translated into the following two equations:

$$0 = (R_r + L_r D)\, I_{qr} + L_m D\, I_{qs} - \omega_o\, \Psi_{dr} \quad (20)$$
$$0 = (R_r + L_r D)\, I_{dr} + L_m D\, I_{ds} + \omega_o\, \Psi_{qr} \quad (21)$$

Based on Eqs. (16, 17, 20 & 21), the d-q equivalent circuit, referred to the stator reference frame, can be drawn as shown in Fig. 4.

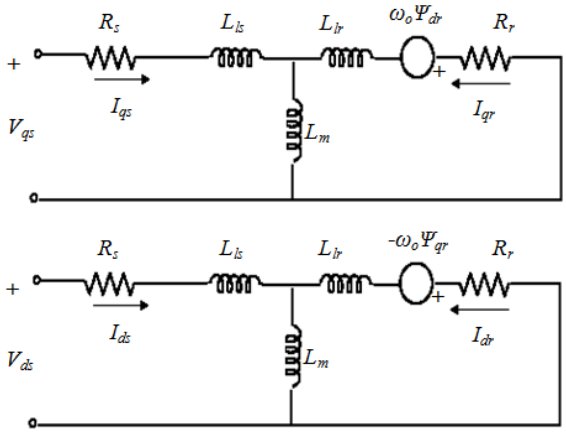

Fig. 4. d-q equivalent circuit referred to the stationary reference frame

Another set of equations, including flux linkage variables, is required to explain the DTC concept. By translating Eqs. (3, 6, 8 & 9) in d-q coordinates on the stator frame, we have the following eight equations:

- Stator and rotor voltage equations:

$$V_{qs} = R_s I_{qs} + D \Psi_{qs} \quad (22)$$
$$V_{ds} = R_s I_{ds} + D \Psi_{ds} \quad (23)$$
$$0 = R_r I_{qr} + D \Psi_{qr} - \omega_o \Psi_{dr} \quad (24)$$
$$0 = R_r I_{dr} + D \Psi_{dr} + \omega_o \Psi_{qr} \quad (25)$$

- Stator and rotor flux linkage equations:

$$\Psi_{qs} = L_s I_{qs} + L_m I_{qr} \quad (26)$$
$$\Psi_{ds} = L_s I_{ds} + L_m I_{dr} \quad (27)$$
$$\Psi_{qr} = L_m I_{qs} + L_r I_{qr} \quad (28)$$
$$\Psi_{dr} = L_m I_{ds} + L_r I_{dr} \quad (29)$$

It will be shown in subsequent sections that the above equations are very useful in the motor model representation and in explaining the concept of the DTC.

2.3 d-q Equivalent Circuit

When a DTC drive controls the induction motor, the control computation is almost written in the stationary d-q frame. Since actual stator variables either to be measured or calculated are all in stationary a-b-c frame, frame transformation should be used in control. A simple transformation from stationary a-b-c quantities to stationary d-q quantities is done by using Eqs.(1, 15), which leads to:

$$Y_{qs} = (2/3)\, Re[\, Y_a(t) + a \,.\, Y_b(t) + a^2 \,.\, Y_c(t) \,] \quad (30)$$
$$Y_{ds} = -(2/3)\, Im[\, Y_a(t) + a \,.\, Y_b(t) + a^2 \,.\, Y_c(t) \,] \quad (31)$$

By using the phasor diagram, Fig. 3, we can rewrite the two previous equations in a simpler form. Note that in our case $\theta_a = 0$ so that:

$$Y_{qs} = Y_a(t) \quad (32)$$
$$Y_{ds} = -(1/\sqrt{3})\, [\, Y_b(t) - Y_c(t) \,] \quad (33)$$

As any motor is 3-wires three-phase load:

$$Y_a(t) + Y_b(t) + Y_c(t) = 0 \quad (34)$$

Therefore Eq.(33) can be rewritten as:

$$Y_{ds} = -(1/\sqrt{3})\, [\, Y_a(t) + 2\, Y_b(t) \,] \quad (35)$$

This is another benefit of using the stator reference frame that we need to measure only two of three-phase

system variables to complete identification of the d-q model.

2.4 Torque Equations

A simple way to obtain the output torque, also called developed or electromagnetic torque, of a three-phase induction motor is to consider the generated electric power associated with the speed voltage term of Fig. 2 as:

$$P_e = (3/2)\ Re\ [\ -j\omega_o\ \boldsymbol{\Psi_r}\ \boldsymbol{I_r}^*\ ] \tag{36}$$

where $\boldsymbol{I_r}^*$ is the complex conjugates of $\boldsymbol{I_r}$, this equation can be translated into:

$$P_e = (3/2)\ \omega_o\ [\ \Psi_{qr}\ I_{dr} - \Psi_{dr}\ I_{qr}\ ] \tag{37}$$

The relationship between the electrical angular frequency ω_o and the mechanical angular speed ω_m, which represents the actual rotor speed in radians per second, is:

$$\omega_o = p\ \ \omega_m \tag{38}$$

where p is the number of machine pole pairs. Also the developed power can be expressed as:

$$P_e = T_e\ .\ \omega_m \tag{39}$$

From the previous three equations, the developed electromagnetic torque can be expressed in d-q form as:

$$T_e = (3/2)\ p\ [\ \Psi_{qr}\ I_{dr} - \Psi_{dr}\ I_{qr}\ ] \tag{40}$$

By substitution from Eqs.(28, 29) in the previous equation we find that:

$$T_e = (3/2)\ p\ [\ \Psi_{ds}\ I_{qs} - \Psi_{qs}\ I_{ds}\ ] \tag{41}$$

The previous equation can be rewritten in space vector form as follow:

$$\boldsymbol{T_e} = \ (3/2)\ p\ \ \boldsymbol{\Psi_s}\ \mathrm{x}\ \boldsymbol{I_s} \tag{42}$$

Other forms of torque equations are applicable. For example, by using Eq.(8) with Eq.(42), we can express the electromagnetic torque in terms of rotor and stator currents as:

$$\boldsymbol{T_e} = \ (3/2)\ p\ L_m\ \ \boldsymbol{I_r}\ \mathrm{x}\ \boldsymbol{I_s} \tag{43}$$

Also by using Eqs.(8, 9) with Eq.(42), we can express the electromagnetic torque in terms of rotor and stator fluxes:

$$\boldsymbol{T_e} = \ (3/2)\ p\ [L_m\ /(L_s\ L_r\ -\ L_m^2\ )]\ \ \boldsymbol{\Psi_r}\ \mathrm{x}\ \ \boldsymbol{\Psi_s} \tag{44}$$

The previous equations are very important in the DTC theory explanation and its analysis. Although the torque expressions above are derived from stationary reference frames, they are valid for other reference frames [3].

3. DIRECT TORQUE CONTROL CONCEPT

Generally, the developed torque by any motor is proportional to the cross-product of the stator flux linkage space vector and the rotor flux linkage space vector [4, 5].

$$\boldsymbol{T_e} = \ k\ \ \boldsymbol{\Psi_r}\ \mathrm{x}\ \ \boldsymbol{\Psi_s} \tag{45}$$

where k is constant. And with reference to Fig. 5:

$$T_e = \ k\ \ \Psi_r\ \ \Psi_s\ \ sin\ \delta \tag{46}$$

which is called the torque production equation. Ψ_r is the rotor flux vector's magnitude, Ψs is the stator flux vector's magnitude and δ is the angle between them, called the torque angle. By comparing Eq.(46) to (44), the three-phase induction motor torque production equation can be written as:

$$T_e = (3/2)\, p\, (L_m / \sigma L_s L_r)\ \Psi_r\ \Psi_s\ \sin\delta \quad (47)$$

where $\sigma = 1 - (L_m^2 / L_s L_r)$; is the leakage coefficient of the motor. It is clear from the torque production equation that the torque can be directly controlled by changing the rotor flux magnitude, stator flux magnitude or the angle between them. In the case of dc motors, they have stationary perpendicular magneto-motive forces. So, the torque angle δ is constant and equal to 90 degrees. The dc motor drives introduced the direct torque control concept, where torque is directly proportional to armature current [4] up to the rated limit. But in the case of three-phase ac motors, the situation is different. The latter has a stator and rotor rotating magnetic fields. The rotor and stator fluxes space vectors rotate along the air gap periphery with an angular speed equal to the three-phase supply angular frequency and with a certain angle δ apart.

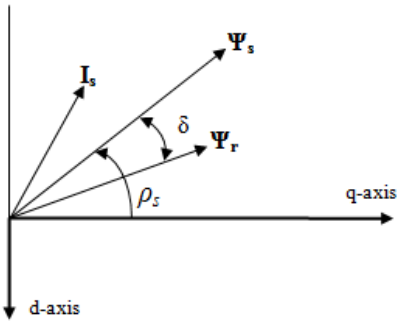

Fig. 5. Rotor and stator flux space vectors

The flux magnitudes are normally kept as constant as possible, and torque is controlled by varying the angle between rotor flux and stator flux vectors [4]. Practically the torque angle is changed by accelerating or decelerating the stator flux vector relative to the rotor flux vector, which can be assumed to be constant during the control action. Thus a quick change in stator flux angle leads to an instantaneous change in torque value. In the case of synchronous motors, the rotor and stator circuits are separated, the rotor flux can't slip the rotor shaft and since the electrical time constant usually is much smaller than the mechanical time constant, the rotating speed of stator flux, with respect to the rotor flux, can be easily changed [6]. In the case of induction motors and from the previous model, the stator flux space vector is related to the rotor flux space vector by the following formula:

$$D\,\boldsymbol{\Psi_r} + [\,(1/\sigma\,\tau_r) - j\omega_o\,]\,\boldsymbol{\Psi_r} = (\,L_m / \sigma\, L_s\, \tau_r\,)\,\boldsymbol{\Psi_s} \quad (48)$$

where $\tau_r = L_r / R_r$ is the rotor time constant. This formula can illustrate the nature of rotor flux dynamic response for a step change in stator flux. It can be obtained by substituting from Eqs.(8, 9) into Eq.(7). In the s-domain, the same expression can be written as:

$$\boldsymbol{\Psi_r} = [\,(L_m / L_s)/(1 + s\,\sigma\,\tau_r)\,]\,\boldsymbol{\Psi_s} \quad (49)$$

This entails that the rotor flux cannot react quickly to changes in the stator flux, as there is a first-order delay relationship between the two fluxes. Thus the rotor flux vector follows the stator flux vector with a time delay related to the time constant $\sigma\,\tau_r$ [7]. Hence, the rotor flux changes slowly compared to the stator flux [5]. Thus, rotor flux is relatively stable and can be assumed constant during quick changes in the stator flux. The assumption of constant rotor flux can be justified when the control action is much faster than the rotor electrical time constant multiplied by the motor leakage coefficient. This determines a quick increase in the angle between the two fluxes vectors and, accordingly, in the torque [8].

3.1 Induction Motor DTC principles

The principle of DTC operation is to select stator voltage vectors according to the differences between the reference stator flux and torque and their actual values [5]. The actual instantaneous torque and stator flux linkage values are calculated from stator variables, namely stator voltage and current, using a closed-loop estimator [8]. Optimal stator voltage vectors are selected to limit the flux and torque errors within predetermined bands for flux and torque hysteresis. The required optimum voltage vectors depend on the stator flux space vector position, available switching vectors and the flux of stator needed and torque. The control scheme aims to

keep the flux linkage constant (within a hysteresis band) [5], ensuring that the magnitude of the rotor space flux vector remains constant as well. The torque is thus controlled by varying the relative angle between the stator and the rotor fluxes. Therefore, the induction motor torque control using DTC depends only on the stator flux vector variation without information about the motor (except for stator resistance to calculate the flux linkage) [5]. To control the stator flux variation, the controller selects one of six voltage vectors by a voltage source inverter, as shown.

3.2 Inverter Voltage Space Vectors

The three-phase voltage source inverter in Fig. 6 illustrates the six available voltage vectors used to control the stator flux and torque in a conventional ac motor, where E is the inverter dc link voltage, and S_a , S_b and S_c are the switching functions of the inverter switches. Each switch may be connected to the dc link negative or positive voltage terminals. Meanwhile, these switches are represented by one status and zero status corresponding to the positive and the negative voltage, respectively [9], which reflects on the motor line voltage. When the inverter supplies a symmetrical ac motor with no neutral connection, the stator space voltage vector can be expressed in terms of the dc link voltage E and the inverter gating signals (S_a , S_b , S_c) by the following equation [10]:

$$V_s = (2/3)\, E\, [S_a + a\, S_b + a^2\, S_c\,] \qquad (50)$$

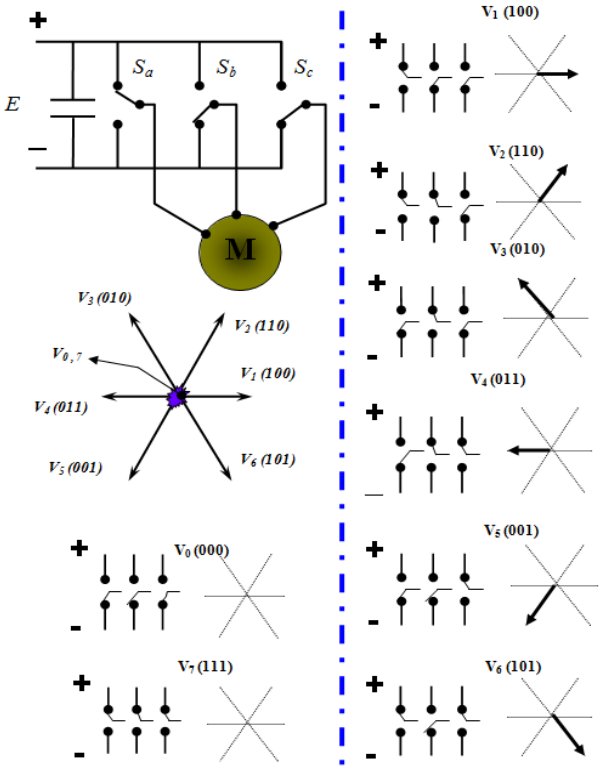

Fig. 6. Available stator space voltage vectors

According to the combination of switching modes, the stator space voltage vectors V_s (S_a , S_b , S_c) are specified in eight distinct vectors. Two of them represent the space zero voltage vectors V_s (1,1,1) and V_s (0,0,0), while the others are nonzero space voltage vectors, e.g. V_s (1,0,0),…., V_s (1,0,1), as shown in Fig. 6 [11, 4]. The space voltage vectors, generated by the inverter and applied to the motor stator winding, control the stator flux linkage space vector movement as shown.

3.3 Stator Flux Movement Control

The stator flux linkage space vector of an induction motor can be expressed in the stationary reference frame by using Eq.(3) as follow:

$$\boldsymbol{\Psi_s} = \int (\boldsymbol{V_s} - R_s \boldsymbol{I_s}) \, dt \qquad (51)$$

During the switching interval Δt, each voltage vector is considered to be constant, and from Eq.(50) the previous equation can be rewritten as:

$$\boldsymbol{\Psi_s} = \boldsymbol{V_s} (S_a , S_b , S_c) \Delta t - \int R_s \boldsymbol{I_s} \, dt + \boldsymbol{\Psi_{so}} \qquad (52)$$

where $\boldsymbol{\Psi_{so}}$ is the initial stator flux linkage at the instant of switching. Except at low voltage levels, the stator resistance drop can be neglected. Thus Eq.(52) can be written as:

$$\Delta\boldsymbol{\Psi_s} = \boldsymbol{V_s} (S_a , S_b , S_c) \Delta t \qquad (53)$$

where, ΔΨs is a vector in the same direction as the stator voltage space vector and scaled by the switching interval. This implies that the end of the stator flux vector will move in the direction of the applied voltage vector. As shown in Fig. 7, the vector ΔΨs or (Vs Δt) has two components. The radial component is responsible for flux magnitude control and the tangential one for flux angle control. Also, they are named amplitude and rotation control components [6].

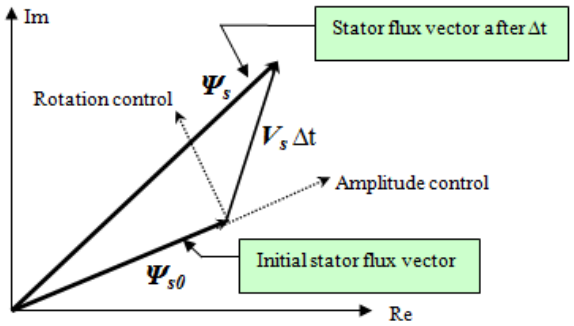

Fig. 7. Stator flux vector radial & tangential movement

4. DTC SCHEME IMPLEMENTATION

A very simple structure characterizes the direct torque control scheme. It mainly consists of two functional blocks [12]:

- Torque and flux estimator (TFE).
- Voltage vector selector (VVS).

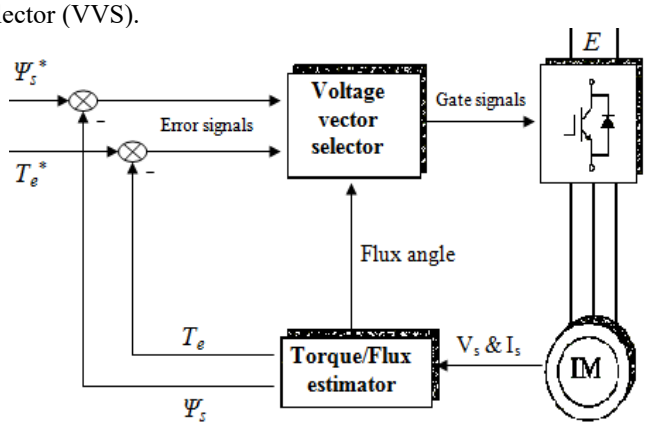

Fig. 8. Basic direct torque control scheme

The core of the DTC scheme is implemented by the basic functional blocks illustrated in Fig. 8 [13].
The torque/flux estimator (TFE) is a very important element in the implementation of the DTC scheme. The DTC scheme needs continuous flux and torque on-line measurements, and there are no sensors that can measure their actual values. The objective of this block is to estimate the actual values of the stator flux linkage space vector (magnitude and angle) referred to the stationary (stator) reference frame and the developed electromagnetic torque level as feedback signals. The dynamic inputs to the torque/flux estimator are the stator voltages and currents.

4.1 Torque/Flux Estimator

To know how the torque/flux estimator calculates its output quantities, we should return to the induction motor mathematical model referred to as the stationary d-q frame and the a-b-c to d-q transformation. Also, let us consider that the analysis of the DTC by digital computers is often done using discrete systems. So, we will define T_s as the sampling interval and k as the sample number, which takes zero or positive integers. The calculation algorithm is as follows:

4.1.1 Calculation of stator direct and quadrature axis voltages:

Two possible methods can be used to calculate the stator terminal voltages of an induction motor in the d-q reference frame fixed to the stator (V_{qs}, V_{ds}). The conventional method is to measure the three-phase motor terminal voltages and transform them to d-q axes voltages [9] by using the transformation equations (32, 35). In this case, two voltage transformers are employed to measure V_{ab} and V_{bc}, which may be a source of error. The second method can be recognized by reviewing equations (50, 15), where the stator voltage space vector and its d-q components are easily calculated from the dc link voltage E and the inverter switches state (S_a , S_b , S_c) at the k^{th} sampling instant as:

$$\boldsymbol{V_s}(k) = (2/3)\, E\, [S_a(k) + a\, S_b(k) + a^2\, S_c(k)] \quad (54)$$

$$V_{qs}(k) = Re[\boldsymbol{V_s}(k)] \;\; \& \;\; V_{ds}(k) = -\, Im[\boldsymbol{V_s}(k)] \quad (55)$$

Thus, no voltage-measuring equipment is needed. At any sampling instant k, the stator voltage space vector is equal to $\boldsymbol{V_0}, \boldsymbol{V_1} \ldots$ or $\boldsymbol{V_7}$. Table 1 shows how we can get the d-q components of each space voltage vector [6], where $V_s = (2/3)\, E$ is the magnitude of the stator voltage space vector.

Table 1: Values of q - d components for the eight stator voltage space vectors

	V_0	V_1	V_2	V_3
V_{qs}	0	V_s	0.5 V_s	-0.5 V_s
V_{ds}	0	0	-0.866 V_s	-0.866 V_s

	V_4	V_5	V_6	V_7
V_{qs}	-V_s	-0.5 V_s	0.5 V_s	0
V_{ds}	0	0.866 V_s	0.866 V_s	0

4.1.2 Calculation of stator direct and quadrature axis currents:

Only two current transformers are needed to measure the motor currents of two stator phases (I_a , I_b). From equations (32, 35), d-q axis stator currents can be calculated at the k^{th} sampling instant as follows:

$$I_{qs}(k) = I_a(k) \quad (56)$$

$$I_{ds}(k) = -(1/\sqrt{3})\, [\, I_a(k) + 2\, I_b(k)\,] \quad (57)$$

4.1.3 Calculation of stator direct and quadrature axis fluxes:

From equations (22, 23), the stator flux space vector d-q components can be calculated at the k^{th} sampling instant as follows [6]:

$$\Psi_{qs}(k) = \Psi_{qs}(k\text{-}1) + [\, V_{qs}(k) - R_s\, I_{qs}(k)]\, T_s \quad (58)$$

$$\Psi_{ds}(k) = \Psi_{ds}(k\text{-}1) + [\, V_{ds}(k) - R_s\, I_{ds}(k)]\, T_s \quad (59)$$

where *(k-1)* is the previous sample. As mentioned before, the stator resistance voltage drop can be neglected, except at low voltage levels which are accompanied by low speed operation. Thus the two previous equations can be written as:

$$\Psi_{qs}(k) = \Psi_{qs}(k-1) + V_{qs}(k)\, T_s \quad (60)$$
$$\Psi_{ds}(k) = \Psi_{ds}(k-1) + V_{ds}(k)\, T_s \quad (61)$$

Then the magnitude and the angle of the stator flux space vector can be calculated as follows:

$$\Psi_s(k) = \sqrt{[\, \Psi_{qs}^2(k) + \Psi_{ds}^2(k)]} \quad (62)$$
$$\rho_s(k) = tan^{-1}[-\Psi_{ds}(k)/\Psi_{qs}(k)] \quad (63)$$

4.1.4 Calculation of the motor developed torque:

Finally, the instantaneous electromagnetic torque can be calculated as follows [6]:

$$T_e(k) = (3/2)\, p\, [\, \Psi_{ds}(k)\, I_{qs}(k) - \Psi_{qs}(k)\, I_{ds}(k)\,] \quad (64)$$

4.2 Voltage Vector Selector

The stator voltage space vector selector, or simply voltage vector selector (VVS), is the DTC head. It receives the torque and flux error signals and the stator flux position angle and properly selects the suitable space voltage vector. The VVS is mainly composed of three blocks:

- Hysteresis comparator
- Space sector locator
- Switching table/logic

Fig. 9 illustrates a schematic diagram of the conventional VVS main components. The switching table accepts logic signals only. So, it receives its binary data from the hysteresis comparator and the space sector locator.

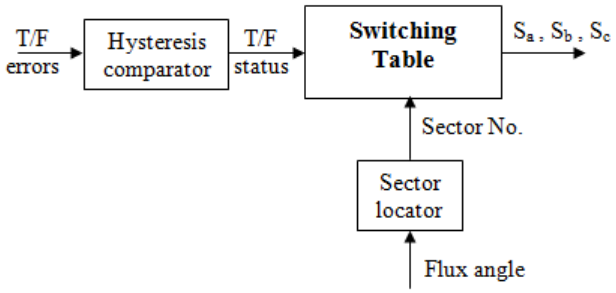

Fig. 9. Schematic diagram of the conventional VVS

4.2.1 Hysteresis Comparator:

The function of the hysteresis comparator is to compare the torque and flux errors with predetermined hysteresis window limits to decide if the torque and flux should be increased or decreased. The output of this block is called torque and flux status. Fig. 10 shows a three-level hysteresis torque error comparator and a two-level hysteresis flux error comparator characteristics [13].

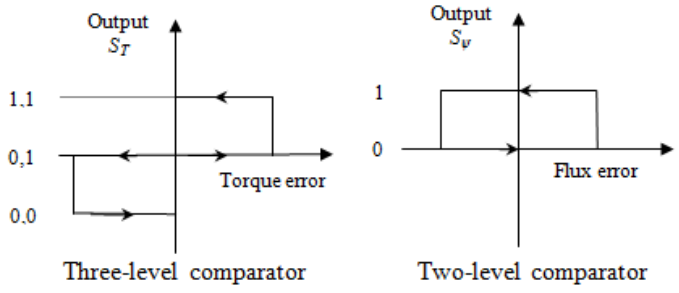

Fig. 10. Torque and flux hysteresis windows

From this figure, it can be seen that the output of the torque error comparator may take the value (0,0), (0,1), or (1,1) depending on the value of the torque error. (0,0) means that torque should be decreased, (1,1) means that torque should be increased. While (0,1) means that torque should be unchanged. Note that (1,0) here is trivial and not used. The output of the flux error comparator may take the value (1) or (0) depending on the value of the flux error. (1) means to increase the flux and (0) means to decrease it. So, the output of the hysteresis comparator is a binary word composed of three bits. This word describes both torque and flux status. The first two bits belong to torque status, and the third bit (the most significant bit) belongs to flux status. Table 2 illustrates binary words corresponding to required torque and flux correction actions.

Table 2: The output binary data of the hysteresis comparator

	increase Torque	No Action	decrease Torque
increase flux	(111)	(101)	(100)
decrease flux	(011)	(001)	(000)

Hysteresis windows define the upper and the lower limits to be used to switch between the different torque and flux statuses. Usually, the value of these limits is chosen to be within ±5 or 2% of the reference value [9]. By nature, the differential hysteresis limits are correlated with the switching frequency of the inverter power solid-state switches. So, the narrower the hysteresis window, the higher the switching frequency will be.

4.2.2 Space Sector Locator:

The function of the space sector locator is to identify the sector on which the stator flux linkage space vector lies at a certain instant. So, its sole input is the flux angle and its output is the flux sector. The space sector locator also expresses the identified sector number as three bits binary word. A three-bit binary word can express eight numbers. Our sector locator uses only six and the remaining two are trivial. Actually, the flux linkage space vector rotates anti-clockwise at synchronous speed. Thus, the flux angle ρ_s varies from 0 to 90 to 180 or –180 to –90 to 0 degrees, and so on.

The scanned 360 degrees are divided into six sectors. Table 3 illustrates these divisions and the corresponding outputs. A possible method to implement the space sector locator is to compare the flux angle to each sector's limits. In Fig. 11, the output of the AND gate will be one if and only if the flux vector lies in sector 1.

Table 3: The output binary data of the sector locator

Flux angle	(-30 to 30)	(30 to 90)	(90 to 150)
Flux sector	S1	S2	S3
Output	(001)	(010)	(011)

Flux angle	(150 to -150)	(-150 to -90)	(-90 to -30)
Flux sector	S4	S5	S6
Output	(100)	(101)	(110)

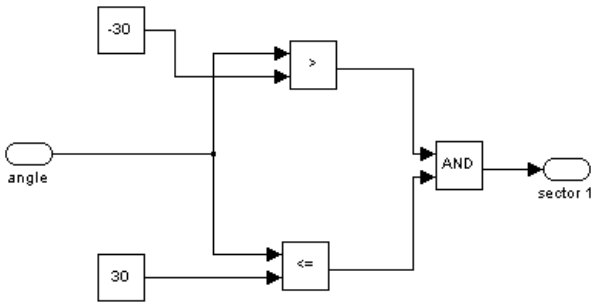

Fig. 11. Elementary space sector identification check

4.2.3 Switching Table/Logic

The switching table or switching logic is the brain of the DTC system. Its function is to select suitable inverter gate signals based on the torque/flux status and the flux vector lying sector. The switching table is simply a two-dimensional look up table. In other words, it can be considered a matrix with 6 rows x 6 columns. Where the hysteresis comparator output determines the row number and the sector locator output determines the column number. Table 4 shows the pre-described switching table [13].
For sure there is a rule to select the nonzero voltage space vector V_1, … , V_6. This rule states that if the stator flux vector is located in sector number (m) in space and the torque status equals (1,1) (i.e. the torque should be increased), there will be two voltage vectors V_{m+1} and V_{m+2} suitable for increasing the torque. The first voltage vector V_{m+1} is used when an increase in the stator flux is also required (i.e. flux status equals 1) but the second voltage vector V_{m+2} is used when a decrease in the stator flux, is required (i.e. flux status equals 0) and so on. Table 4 also summarizes the selection rules of the switching logic [14]. "+1" means one step forward and "-1" means one step backwards from 1 to 6 to 1 as a closed cycle.

Table 4: Optimum switching table & appropriate voltage vector selection rules

	S_1	S_2	S_3	S_4	S_5	S_6
Fi & Ti	V_2	V_3	V_4	V_5	V_6	V_1
NA	V_0	V_7	V_0	V_7	V_0	V_7
Fi & Td	V_6	V_1	V_2	V_3	V_4	V_5
Fd & Ti	V_3	V_4	V_5	V_6	V_1	V_2
NA	V_7	V_0	V_7	V_0	V_7	V_0
Fd & Td	V_5	V_6	V_1	V_2	V_3	V_4

	Ti	NA	Td
Fi	V_{m+1}	V_0 or V_7	V_{m-1}
Fd	V_{m+2}	V_0 or V_7	V_{m-2}

F= flux, T=torque, i= increase, d=decrease, NA= no action & S=sector

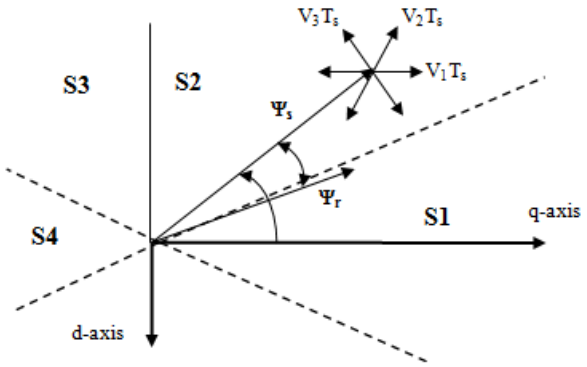

Fig. 12. Voltage vector selection for a flux vector lying in sector 2

Fig. 12 illustrates how the previous rules can apply to a flux vector in sector 2. In case of no action required in torque (torque error is within acceptable limits), a zero voltage vector is selected without consideration of the flux status in order to reduce the torque ripples. To apply a zero voltage vector, V_0 or V_7 can be selected; however, it is found that alternating between them may cause better performance for inverter circuit. This is illustrated in Table 4 too.

5. DTC OPERATION

Referring to Fig. 8, the DTC operation starts with the feedback signals (V_s & I_s), which are fed to the torque/flux estimator (TFE). The TFE calculates the torque and flux magnitude actual values and the flux position angle. Actual flux and torque are compared with the reference values and the errors are fed to the voltage vector selector (VVS).

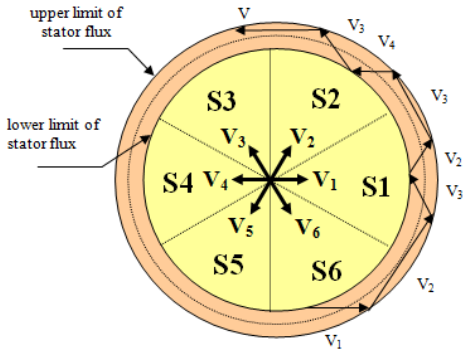

Fig. 13. The stator flux vector tip locus during the DTC operation

The latter receives torque and flux errors and the flux angle. According to error values and angle, the VVS selects the suitable inverter switching state. All of these actions happen outside the motor and are translated inside the motor as a continuous stator flux linkage motion. The latter motion is complex somewhere. It should rotate at a synchronous speed with respect to the stator. Also, it should always be in relative rotation with respect to the rotor flux linkage to increase and decrease the torque angle δ. Finally, its magnitude moves between the lower and upper limits of the flux hysteresis window.
Fig. 13 shows how the appropriate step-by-step voltage vector selections drive the stator flux vector during its motion. In steady state conditions, the stator flux vector draws a circular locus, except for ripples, due to the switching effect [8]. This simple approach achieves a quick torque response. However, undesirable ripples in torque and current accompany the steady state performance [13].

6. SPEED CONTROL BASED ON DTC

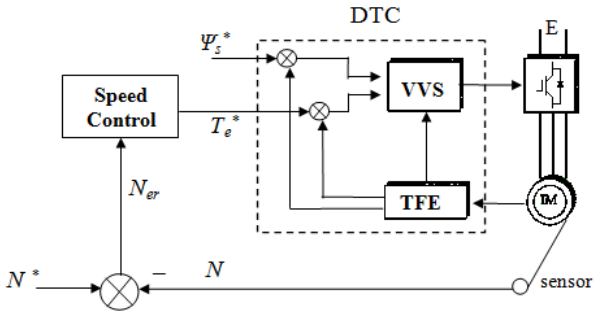

Fig. 14. Speed control based on the DTC scheme

Fig. 14 illustrates the torque control loop (primary loop) and the speed control loop (secondary loop) which is based on DTC. In this system, the speed reference input is compared to the actual speed feedback obtained from a speed sensor. The speed error signal is the input to the speed control block. The resulting output signal from the speed control becomes the torque reference for the DTC subsystem. From here, it is clear that the speed control generates the torque command, i.e., the speed error value determines the torque reference. This leads to the fact that torque control and speed control cannot be achieved at the same time. However, and as mentioned, industrial applications need torque control as well as speed control. In order to swap between torque control and speed control, a torque reference selector is added to the previous system. Fig. 15 illustrates how the torque reference is selected based on the control mode input.

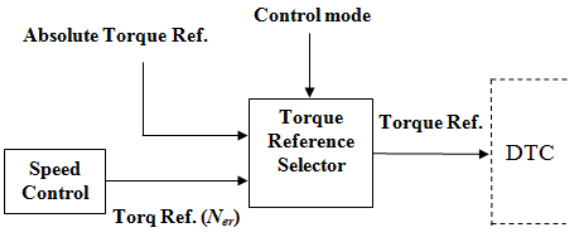

Fig. 15. Torque reference selector schematic diagram

The torque reference selector has three inputs: torque reference as a function of the speed error, absolute torque reference and a control mode. The control mode input can take two values: “low” or “high” (0,1). “0” leads to torque control mode. “1” leads to speed control mode. Only the absolute torque reference is used when the drive works as a torque control. When the drive works as a speed control, only the torque reference, which depends on the speed error, is used. These two references are never used simultaneously [4].

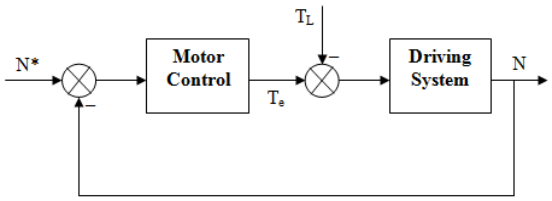

Fig. 16. Block diagram of the motoring system

The output torque reference or demand is processed by the DTC as described before. In fact, the speed control mode also can be divided into two modes of operation: tracking mode and regulating mode. As shown in Fig. 16, the speed reference is the primary input to the system. When the speed reference is changed, the speed controller works to make the motor speed follow the reference speed as possible (tracking). When a load variation occurs (disturbance input), the speed controller resists any probable changes in the motor speed due to this load torque variation (regulating).

7. CONCLUSION

A complete study for three-phase induction motor DTC is presented. A suitable mathematical model choice simplifies the motor's analysis and control. The speed control can be based on the torque control. The switching between torque control and speed control is possible, but both cannot be achieved simultaneously.

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